Name:_____Class:_____ Date: _____

Two-Dimensional Motion and Vectors

Problem C

ADDING VECTORS ALGEBRAICALLY

PROBLEM

The southernmost point in the United States is called South Point, and is located at the southern tip of the large island of Hawaii. A plane designed to take off and land in water leaves South Point and flies to Honolulu, on the island of Oahu, in three separate stages. The plane first flies 83.0 km at 22.0° west of north from South Point to Kailua Kona, Hawaii. The plane then flies 146 km at 21.0° west of north from Kailua Kona to Kahului, on the island of Maui. Finally, the plane flies 152 km at 17.5° north of west from Kahului to Honolulu. What is the plane's resultant displacement?

SOLUTION

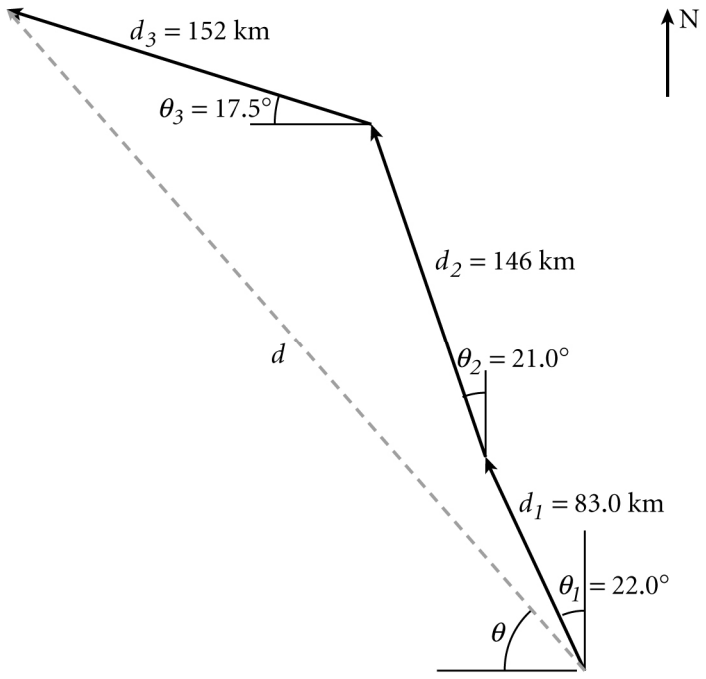
1. **DEFINE**

Given:

$d_1 = 83.0$ km	$\theta_1 = 22.0°$ west of north
$d_2 = 146$ km	$\theta_2 = 21.0°$ west of north
$d_3 = 152$ km	$\theta_3 = 17.5°$ north of west

Unknown: $d = ?$ $\theta = ?$

Diagram:



2. **PLAN** **Choose the equation(s) or situation:** Express the components of each vector in terms of sine or cosine functions.

$\Delta x_1 = d_1 (\sin \theta_1)$ $\Delta y_1 = d_1 (\cos \theta_1)$

$\Delta x_2 = d_2 (\sin \theta_2)$ $\Delta y_2 = d_2 (\cos \theta_2)$

$\Delta x_3 = d_3 (\cos \theta_3)$ $\Delta y_3 = d_3 (\sin \theta_3)$

Note that the angles θ_1 and θ_2 are with respect to the y axis (north), and so the x components are in terms of sin θ. Write the equations for Δx_{tot} and Δy_{tot}, the components of the total displacement.

$$\Delta x_{tot} = \Delta x_1 + \Delta x_2 + \Delta x_3$$
$$= d_1 (\sin \theta_1) + d_2(\sin \theta_2) + d_3 (\cos \theta_3)$$
$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 + \Delta y_3$$
$$= d_1 (\cos \theta_1) + d_2(\cos \theta_2) + d_3 (\sin \theta_3)$$

Use the components of the total displacement, the Pythagorean theorem, and the tangent function to calculate the total displacement.

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2}$$
$$\theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right)$$

3. **CALCULATE** **Substitute the values into the equation(s) and solve:**

$$\Delta x_{tot} = (83.0 \text{ km})(\sin 22.0°) + (146 \text{ km})(\sin 21.0°) + (152 \text{ km})(\cos 17.5°)$$
$$= 31.1 \text{ km} + 52.3 \text{ km} + 145 \text{ km}$$
$$= 228 \text{ km}$$
$$\Delta y_{tot} = (83.0 \text{ km})(\cos 22.0°) + (146 \text{ km})(\cos 21.0°) + (152 \text{ km})(\sin 17.5°)$$
$$= 259 \text{ km}$$
$$d = \sqrt{(228 \text{ km})^2 + (259 \text{ km})^2} = \sqrt{5.20 \times 10^4 \text{ km}^2 + 6.71 \times 10^4 \text{ km}^2} = \sqrt{11.91 \times 10^4 \text{ km}^2}$$
$$= \boxed{345.1 \text{ km}}$$
$$\theta = \tan^{-1}\left(\frac{259 \text{ km}}{228 \text{ km}}\right) = \boxed{48.6° \text{ north of west}}$$

4. **EVALUATE** If the diagram is drawn to scale, compare the calculated results to the drawing. The length of the drawn resultant is fairly close to the scaled magnitude for d, while the angle appears to be slightly greater than 45°.

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ADDITIONAL PRACTICE

1. U.S. Highway 212 extends 55 km at 37° north of east between Newell and Mud Butte, South Dakota. It then continues for 66 km nearly due east from Mud Butte to Faith, South Dakota. If you drive along this part of U.S. Highway 212, what will be your total displacement?
2. Wrigley Field is one of only three original major-league baseball fields that are still in use today. Suppose you want to drive to Wrigley Field from the corner of 55th Street and Woodlawn Avenue, about 14 miles south of Wrigley Field. Although not the fastest or most direct route, the most straightforward way to reach Wrigley Field is to drive 4.1 km west on 55th Street to Halsted Street, then turn north and drive 17.3 km on Halsted until you reach Clark Street. Turning on Clark, you will reach Wrigley Field after traveling 1.2 km at an angle of 24.6° west of north. What is your resultant displacement?
3. A bullet traveling 850 m ricochets from a rock. The bullet travels another 640 m, but at an angle of 36° from its previous forward motion. What is the resultant displacement of the bullet?
4. The cable car system in San Francisco is the last of its kind that is still in use in the United States. It was originally designed to transport large numbers of people up the steep hills on which parts of the city are built. If you ride seven blocks on the Powell Street cable car from the terminal at Market Street to Pine Street, you will travel 2.00×10^2 m on level ground, then 3.00×10^2 m at an incline of 3.0° to the horizontal, and finally 2.00×10^2 m at 8.8° to the horizontal. What will be your resultant displacement?
5. An Arctic tern flying to Antarctica encounters a storm. The tern changes direction to fly around the storm. If the tern flies 46 km at 15° south of east, 22 km at 13° east of south, and finally 14 km at 14° west of south, what is the tern's resultant displacement?
6. A technique used to change the direction of space probes, as well as to give them additional speed, is to use the gravitational pull of nearby planet. This technique was first used with the Voyager probes. Voyager 2 had traveled about 6.3×10^8 km when it reached Jupiter. Jupiter's gravity changed Voyager's direction by 68°. The probe then traveled about 9.4×10^8 km when it reached Saturn, and its direction was changed by 94°. Voyager 2 was now redirected; it encountered Uranus after traveling 3.4×10^9 km from Saturn. Use this information to calculate the resultant displacement of Voyager 2 as it traveled from Earth to Uranus.

7. The city of Amsterdam, in the Netherlands, has several canals that connect different sections of the city. Suppose you take a barge trip to the harbor, starting at a point near the northwest corner of the Vondelpark. You would sail 2.50×10^3 m at 58.5° north of east, 375 m at 21.8° north of east, and 875 m at 21.5° east of north. What would be your resultant displacement?
8. The elevated train, or "L," in Chicago is a major source for mass transit in that city. One of the lines extends from Jefferson Park, in the northwest part of town, to the Clark Street station downtown. The route of this line runs 5.0 km at 36.9° south of east, 1.5 km due south, 8.5 km at 42.2° south of east, and 0.8 km due east. What is the resultant displacement of an "L" train from Jefferson Park to Clark Street?
9. A billiard table is positioned with its long side parallel to north. A cue ball is then shot so that it travels 1.41 m at an angle of 45.0° west of north, is deflected by the table's left side, and continues to move 1.98 m east of north at an angle of 45.0°. The ball is then deflected by the table's right side, so that it moves 0.42 m west of north at an angle of 45.0°. After a reflection on the north end of the table, the ball travels 1.56 m at an angle of 45.0° south of west. Determine the resultant displacement of the cue ball.
10. Hurricane Iniki was the most destructive cyclone to have crossed the Hawaiian Islands in the twentieth century. It's path was also unusual: it moved south of the islands for 790 km at an angle of 18° north of west, then moved due west for 150 km, turned north and continued for 470 km, and finally turned back 15° east of north and moved 240 km to cross the island of Kauai. What was the resultant displacement of Hurricane Iniki?