

## Two-Dimensional Motion and Vectors

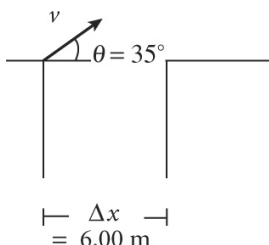
**Problem E****PROJECTILES LAUNCHED AT AN ANGLE****PROBLEM**

The narrowest strait on Earth is Seil Sound in Scotland, which lies between the mainland and the island of Seil. The strait is only about 6.0 m wide. Suppose an athlete wanting to jump “over the sea” leaps at an angle of  $35^\circ$  with respect to the horizontal. What is the minimum initial speed that would allow the athlete to clear the gap? Neglect air resistance.

**SOLUTION****1. DEFINE**

**Given:**  $\Delta x = 6.0 \text{ m}$   
 $\theta = 35^\circ$   
 $a_y = -g = -9.81 \text{ m/s}^2$

**Unknown:**  $v_i = ?$

**2. PLAN Diagram:**

**Choose the equation(s) or situation:** The horizontal component of the athlete’s velocity,  $v_x$ , is equal to the initial speed multiplied by the cosine of the angle,  $\theta$ , which is equal to the magnitude of the horizontal displacement,  $\Delta x$ , divided by the time interval required for the complete jump.

$$v_x = v_i \cos \theta = \frac{\Delta x}{\Delta t}$$

At the midpoint of the jump, the vertical component of the athlete’s velocity,  $v_y$ , which is the upward vertical component of the initial velocity,  $v_i \sin \theta$ , plus the component of velocity due to free-fall acceleration, equals zero. The time required for this to occur is half the time necessary for the total jump.

$$v_y = v_i \sin \theta + a_y \left( \frac{\Delta t}{2} \right) = 0$$

$$v_i \sin \theta = \frac{-a_y \Delta t}{2}$$

**Rearrange the equation(s) to isolate the unknown(s):** Express  $\Delta t$  in the second equation in terms of the displacement and velocity component in the first equation.

$$v_i \sin \theta = \frac{-a_y}{2} \left( \frac{\Delta x}{v_i \cos \theta} \right)$$

$$v_i^2 = \frac{-a_y \Delta x}{2 \sin \theta \cos \theta}$$

$$v_i = \sqrt{\frac{-a_y \Delta x}{2 \sin \theta \cos \theta}}$$

3. **CALCULATE** Substitute the values into the equation(s) and solve: Select the positive root for  $v_i$ .

$$v_i = \sqrt{\frac{-(-9.81 \text{ m/s}^2)(6.0 \text{ m})}{(2)(\sin 35^\circ)(\cos 35^\circ)}} = 7.9 \text{ m/s}$$

4. **EVALUATE** By substituting the value for  $v_i$  into the original equations, you can determine the time for the jump to be completed, which is 0.92 s. From this, the height of the jump is found to equal 1.0 m.

### ADDITIONAL PRACTICE

- In 1993, Wayne Brian threw a spear a record distance of 201.24 m. (This is not an official sport record because a special device was used to “elongate” Brian’s hand.) Suppose Brian threw the spear at a  $35.0^\circ$  angle with respect to the horizontal. What was the initial speed of the spear?
- April Moon set a record in flight shooting (a variety of long-distance archery). In 1981 in Utah, she sent an arrow a horizontal distance of  $9.50 \times 10^2$  m. What was the speed of the arrow at the top of the flight if the arrow was launched at an angle of  $45.0^\circ$  with respect to the horizontal?
- In 1989 during overtime in a high school basketball game in Erie, Pennsylvania, Chris Eddy threw a basketball a distance of 27.5 m to score and win the game. If the shot was made at a  $50.0^\circ$  angle above the horizontal, what was the initial speed of the ball?
- In 1978, Geoff Capes of the United Kingdom won a competition for throwing 5 lb bricks; he threw one brick a distance of 44.0 m. Suppose the brick left Capes’ hand at an angle of  $45.0^\circ$  with respect to the horizontal.
  - What was the initial speed of the brick?
  - What was the maximum height reached by the brick?
  - If Capes threw the brick straight up with the speed found in (a), what would be the maximum height the brick could achieve?
- In 1991, Doug Danger rode a motorcycle to jump a horizontal distance of 76.5 m. Find the maximum height of the jump if his angle with respect to the ground at the beginning of the jump was  $12.0^\circ$ .
- Michael Hout of Ohio can run 110.0 meter hurdles in 18.9 s at an average speed of 5.82 m/s. What makes this interesting is that he juggles three balls as he runs the distance. Suppose Hout throws a ball up and forward at twice his running speed and just catches it at the same level. At what angle,  $\theta$ , must the ball be thrown? (Hint: Consider horizontal displacements for Hout and the ball.)

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

7. A scared kangaroo once cleared a fence by jumping with a speed of 8.42 m/s at an angle of  $55.2^\circ$  with respect to the ground. If the jump lasted 1.40 s, how high was the fence? What was the kangaroo's horizontal displacement?
8. Measurements made in 1910 indicate that the common flea is an impressive jumper, given its size. Assume that a flea's initial speed is 2.2 m/s, and that it leaps at an angle of  $21^\circ$  with respect to the horizontal. If the jump lasts 0.16 s, what is the magnitude of the flea's horizontal displacement? How high does the flea jump?