Circuits and Circuit Elements

Problem B

RESISTORS IN PARALLEL
PROBLEM
A light bulb in a camper's flashlight is labeled 2.4 V, 0.70 A. Find the equivalent resistance and the current if three of these light bulbs are connected in parallel to a standard C size 1.5 V battery.

SOLUTION
1. **DEFINE**

Given:

$\Delta V_1 = 2.4$ V $\qquad I_1 = 0.70$ A

$\Delta V_2 = 2.4$ V $\qquad I_2 = 0.70$ A

$\Delta V_3 = 2.4$ V $\qquad I_3 = 0.70$ A

$\Delta V = 1.5$ V

Unknown: $R_{eq} = ?$ $\qquad I = ?$

2. **PLAN** **Choose the equation(s) or situation:** Because the resistors (bulbs) are in parallel, the equivalent resistance can be calculated from the equation for resistors in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

To calculate the individual resistances, use the definition of resistance.

$$\Delta V_n = I_n R_n$$

The following form of the equation can be used to calculate the current.

$$\Delta V = I R_{eq}$$

Rearrange the equation(s) to isolate the unknown(s):

$$R_n = \frac{\Delta V_n}{I_n} \qquad I = \frac{\Delta V}{R_{eq}}$$

3. **CALCULATE** **Substitute the values into the equation(s) and solve:**

$$R_1 = \frac{\Delta V_1}{I_1} = \frac{(2.4 \text{ V})}{(0.70 \text{ A})} = 3.4 \ \Omega$$

$$R_2 = \frac{\Delta V_2}{I_2} = \frac{(2.4 \text{ V})}{(0.70 \text{ A})} = 3.4 \ \Omega$$

$$R_3 = \frac{\Delta V_3}{I_3} = \frac{(2.4 \text{ V})}{(0.70 \text{ A})} = 3.4 \ \Omega$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{3}{(3.4 \ \Omega)} = \frac{0.88}{1 \ \Omega}$$

$$R_{eq} = 1.1 \ \Omega$$

$$I = \frac{(1.5 \text{ V})}{(1.1 \ \Omega)} = 1.4 \text{ V}$$

4. **EVALUATE**   For resistors connected in parallel, the equivalent resistance should be less than the smallest resistance in the circuit.

$$1.1 \ \Omega < 3.4 \ \Omega$$

ADDITIONAL PRACTICE

1. A certain full-range loudspeaker has a maximum resistance of 32 Ω at 45 Hz, a resistance of 5.0 Ω for most audible frequencies, and a resistance of only 1.8 Ω at 20 kHz. Consider three resistors with resistances of 1.8 Ω, 5.0 Ω, and 32 Ω. Find the equivalent resistance if they are connected in parallel.

2. The Large Electron Positron ring, near Geneva, Switzerland, is one of the biggest scientific instruments on Earth. The circumference of the ring is 27 km. A copper wire with this length and a cross-sectional area of 1 mm^2 will have a resistance of about 450 Ω. Consider a parallel connection of three resistors with resistances equal to 1.0, 2.0, and 0.50 times the resistance of the copper wire, respectively. What is the equivalent resistance?

3. Cars on the Katoomba Scenic Railway are pulled along by winding cables, and at one point, they move along a 310 m span that makes an angle of 51° with the horizontal. A 310 m steel cable that is 4 cm thick would have an estimated resistance of 2.48×10^{-2} Ω. An equivalent resistance of 6.00×10^{-3} Ω can be obtained if two resistors, one having the same resistance as the steel cable, are connected in parallel. Find the resistance of the second resistor.

4. In 1992 in Atlanta, 1 724 000 United States quarters were placed side by side in a straight line. Suppose these quarters were stacked to form a cylindrical tower. If the influence of the air gaps between coins is negligible, the resistance of the tower can be estimated easily. Find the resistance if the parallel connection of four resistors that have resistances equal to exactly 1, 3, 7, and 11 times the tower's resistance yields an equivalent resistance of 6.38×10^{-2} Ω.

5. The largest piece of gold ever found had a mass of about 70 kg. If you were to draw this mass of gold out into a thin wire with a cross-sectional area of 2.0 mm^2, the wire would have a length of 1813 km. The wire would also have a resistance per unit length of 1.22×10^{-2} Ω/m.
    a. What is the resistance of the wire?
    b. Suppose the wire were cut into pieces having resistance of exactly 1/2, 1/4, 1/5, and 1/20 of the wire's resistance, respectively. If these pieces are reconnected in parallel, what is the equivalent resistance of the four pieces?

6. A powerful cordless drill uses a 14.4 V battery to deliver 225 W of power. Treating the drill as a resistor, find its resistance. If a single 14.4 V battery is connected to four "drill" resistors that are connected in parallel, what are the equivalent resistance and the battery current?

7. The total length of the telephone wires in the Pentagon is 3.22×10^5 km. Suppose these wires have a resistance of 1.0×10^{-2} Ω/m. If all the wires are cut into 1.00×10^{-3} km pieces and all pieces are connected in parallel to a AA battery ($\Delta V = 150$ V), what would the current through the wires be? Assume that a AA battery can sustain this current.